

TRIGONOMETRIC FUNCTIONS

Exercise 3.1 Page No: 54

1. Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) $-47^\circ 30'$ (iii) 240° (iv) 520°

Solution:

(i) 25°

Here $180^\circ = \pi$ radian

It can be written as

$$25^\circ = \frac{\pi}{180} \times 25 \text{ radian}$$

So we get

$$= \frac{5\pi}{36} \text{ radian}$$

(ii) $-47^\circ 30'$

Here $1^\circ = 60'$

It can be written as

$$-47^\circ 30' = -47\frac{1}{2} \text{ degree}$$

So we get

$$= \frac{-95}{2} \text{ degree}$$

Here $180^\circ = \pi$ radian

$$\frac{-95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian}$$

It can be written as

$$= \left(\frac{-19}{36 \times 2}\right) \pi \text{ radian} = \frac{-19}{72} \pi \text{ radian}$$

We get

$$-47^\circ 30' = \frac{-19}{72} \pi \text{ radian}$$

(iii) 240°

Here $180^\circ = \pi$ radian

It can be written as

$$240^\circ = \frac{\pi}{180} \times 240 \text{ radian}$$

So we get

$$= \frac{4}{3} \pi \text{ radian}$$

(iv) 520°

Here $180^\circ = \pi$ radian

It can be written as

$$520^\circ = \frac{\pi}{180} \times 520 \text{ radian}$$

So we get

$$= \frac{26\pi}{9} \text{ radian}$$

**2. Find the degree measures corresponding to the following radian measures
(Use $\pi = 22/7$)**

(i) $11/16$

(ii) -4

(iii) $5\pi/3$

(iv) $7\pi/6$

Solution:

(i) $11/16$

Here π radian $= 180^\circ$

$$\frac{11}{16} \text{ radian} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree}$$

We can write it as

$$= \frac{45 \times 11}{\pi \times 4} \text{ deg ree}$$

So we get

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree}$$

$$= \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

Take $1^\circ = 60'$

$$= 39^\circ + \frac{3 \times 60}{8} \text{ min utes}$$

We get

$$= 39^\circ + 22' + \frac{1}{2} \text{ min utes}$$

Consider $1' = 60''$

$$= 39^\circ 22' 30''$$

(ii) -4

Here π radian $= 180^\circ$

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree}$$

We can write it as

$$= \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

By further calculation

$$= \frac{-2520}{11} \text{ deg ree} = -229 \frac{1}{11} \text{ deg ree}$$

Take $1^\circ = 60'$

$$= -229^\circ + \frac{1 \times 60}{11} \text{ min utes}$$

So we get

$$= -229^\circ + 5' + \frac{5}{11} \text{ min utes}$$

Again $1' = 60''$

$$= -229^\circ 5' 27''$$

(iii) $5\pi/3$

Here $\pi \text{ radian} = 180^\circ$

$$\frac{5\pi}{3} \text{ radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree}$$

We get

$$= 300^\circ$$

(iv) $7\pi/6$

Here $\pi \text{ radian} = 180^\circ$

$$\frac{7\pi}{6} \text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6}$$

We get

$$= 210^\circ$$

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

It is given that

No. of revolutions made by the wheel in

$$1 \text{ minute} = 360$$

$$1 \text{ second} = 360/60 = 6$$

We know that

The wheel turns an angle of 2π radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of $6 \times 2\pi \text{ radian} = 12\pi \text{ radian}$

Therefore, in one second, the wheel turns an angle of 12π radian.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = 22/7$).

Solution:

Consider a circle of radius r unit with l unit as the arc length which subtends an angle θ radian at the centre

$$\theta = l/r$$

Here $r = 100$ cm, $l = 22$ cm

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree}$$

It can be written as

$$= \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$

$$= \frac{126}{10} \text{ deg ree}$$

So we get

$$= 12 \frac{3}{5} \text{ deg ree}$$

Here $1^\circ = 60'$

$$= 12^\circ 36'$$

Therefore, the required angle is $12^\circ 36'$.

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

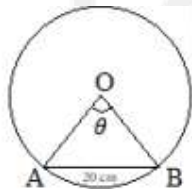
Solution:

The dimensions of the circle are

Diameter = 40 cm

Radius = $40/2 = 20$ cm

Consider AB be as the chord of the circle i.e. length = 20 cm



In $\triangle OAB$,

Radius of circle = $OA = OB = 20$ cm

Similarly $AB = 20$ cm

Hence, $\triangle OAB$ is an equilateral triangle.

$$\theta = 60^\circ = \pi/3 \text{ radian}$$

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre

We get $\theta = l/r$

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Therefore, the length of the minor arc of the chord is $20\pi/3$ cm.

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Consider r_1 and r_2 as the radii of the two circles.

Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 and an arc of length

l subtend an angle of 75° at the centre of the circle of radius r_2 .

Here $60^\circ = \pi/3$ radian and $75^\circ = 5\pi/12$ radian

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre

We get

$$\theta = l/r \text{ or } l = r\theta$$

We know that

$$l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

By equating both we get

$$\frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

On further calculation

$$r_1 = \frac{r_2 5}{4}$$

So we get

$$\frac{r_1}{r_2} = \frac{5}{4}$$

Therefore, the ratio of the radii is 5: 4.

7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

In a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = l/r$

We know that $r = 75$ cm

(i) $l = 10$ cm

So we get

$$\theta = 10/75 \text{ radian}$$

By further simplification

$$\theta = 2/15 \text{ radian}$$

(ii) $l = 15$ cm

So we get

$$\theta = 15/75 \text{ radian}$$

By further simplification

$$\theta = 1/5 \text{ radian}$$

(iii) $l = 21$ cm

So we get

$$\theta = 21/75 \text{ radian}$$

By further simplification

$$\theta = 7/25 \text{ radian}$$

Exercise 3.2 page: 63

Find the values of other five trigonometric functions in Exercises 1 to 5.

1. $\cos x = -1/2$, x lies in third quadrant.

Solution:

It is given that

$$\cos x = -1/2$$

$$\sec x = 1/\cos x$$

Substituting the values

$$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

Consider

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\sin^2 x = 1 - \cos^2 x$$

Substituting the values

$$\sin^2 x = 1 - (-1/2)^2$$

$$\sin^2 x = 1 - 1/4 = 3/4$$

$$\sin^2 x = \pm \sqrt{3}/2$$

Here x lies in the third quadrant so the value of $\sin x$ will be negative

$$\sin x = -\sqrt{3}/2$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2. $\sin x = 3/5$, x lies in second quadrant.

Solution:

It is given that

$$\sin x = 3/5$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

We know that

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\cos^2 x = 1 - \sin^2 x$$

Substituting the values

$$\cos^2 x = 1 - (3/5)^2$$

$$\cos^2 x = 1 - 9/25$$

$$\cos^2 x = 16/25$$

$$\cos x = \pm 4/5$$

Here x lies in the second quadrant so the value of cos x will be negative

$$\cos x = -4/5$$

We can write it as

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

Here

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

3. $\cot x = 3/4$, x lies in third quadrant.

Solution:

It is given that

$$\cot x = 3/4$$

We can write it as

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

We know that

$$1 + \tan^2 x = \sec^2 x$$

We can write it as

$$1 + (4/3)^2 = \sec^2 x$$

Substituting the values

$$1 + 16/9 = \sec^2 x$$

$$\cos^2 x = 25/9$$

$$\sec x = \pm 5/3$$

Here x lies in the third quadrant so the value of $\sec x$ will be negative

$$\sec x = -5/3$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{4}{3} = \frac{\sin x}{\left(-\frac{3}{5}\right)}$$

By further calculation

$$\sin x = \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4. $\sec x = 13/5$, x lies in fourth quadrant.

Solution:

It is given that

$$\sec x = 13/5$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

We know that

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\sin^2 x = 1 - \cos^2 x$$

Substituting the values

$$\sin^2 x = 1 - (5/13)^2$$

$$\sin^2 x = 1 - 25/169 = 144/169$$

$$\sin^2 x = \pm 12/13$$

Here x lies in the fourth quadrant so the value of $\sin x$ will be negative

$$\sin x = -12/13$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

5. $\tan x = -5/12$, x lies in second quadrant.

Solution:

It is given that

$$\tan x = -5/12$$

We can write it as

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

We know that

$$1 + \tan^2 x = \sec^2 x$$

We can write it as

$$1 + (-5/12)^2 = \sec^2 x$$

Substituting the values

$$1 + 25/144 = \sec^2 x$$

$$\sec^2 x = 169/144$$

$$\sec x = \pm 13/12$$

Here x lies in the second quadrant so the value of $\sec x$ will be negative

$$\sec x = -13/12$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

By further calculation

$$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the values of the trigonometric functions in Exercises 6 to 10.

6. $\sin 765^\circ$

Solution:

We know that values of $\sin x$ repeat after an interval of 2π or 360°

So we get

$$\sin 765^\circ = \sin (2 \times 360^\circ + 45^\circ)$$

By further calculation

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

7. $\operatorname{cosec} (-1410^\circ)$

Solution:

We know that values of $\operatorname{cosec} x$ repeat after an interval of 2π or 360°

So we get

$$\operatorname{cosec} (-1410^\circ) = \operatorname{cosec} (-1410^\circ + 4 \times 360^\circ)$$

By further calculation

$$= \operatorname{cosec} (-1410^\circ + 1440^\circ)$$

$$= \operatorname{cosec} 30^\circ = 2$$

8. $\tan \frac{19\pi}{3}$

Solution:

We know that values of $\tan x$ repeat after an interval of π or 180°

So we get

$$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

By further calculation

$$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3}$$

We get

$$= \tan 60^\circ$$

$$= \sqrt{3}$$

$$9. \sin\left(-\frac{11\pi}{3}\right)$$

Solution:

We know that values of $\sin x$ repeat after an interval of 2π or 360°

So we get

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right)$$

By further calculation

$$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$10. \cot\left(-\frac{15\pi}{4}\right)$$

Solution:

We know that values of $\tan x$ repeat after an interval of π or 180°

So we get

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$$

By further calculation

$$= \cot \frac{\pi}{4} = 1$$

<https://loyaleducation.org>

Exercise 3.3 page: 73

Prove that:

1.

$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$$

So we get

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

By further calculation

$$= 1/4 + 1/4 - 1$$

$$= -1/2$$

$$= \text{RHS}$$

2.

$$2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$$

Solution:

Consider

$$\text{L.H.S.} = 2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3}$$

By further calculation

$$= 2 \left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6}\right) \left(\frac{1}{2}\right)^2$$

It can be written as

$$= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6}\right)^2 \left(\frac{1}{4}\right)$$

So we get

$$= \frac{1}{2} + (-2)^2 \left(\frac{1}{4}\right)$$

Here

$$= 1/2 + 4/4$$

$$= 1/2 + 1$$

$$= 3/2$$

$$= \text{RHS}$$

3.

$$\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$$

Solution:

Consider

$$\text{L.H.S.} = \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6}$$

So we get

$$= (\sqrt{3})^2 + \operatorname{cosec} \left(\pi - \frac{\pi}{6} \right) + 3 \left(\frac{1}{\sqrt{3}} \right)^2$$

By further calculation

$$= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3}$$

We get

$$= 3 + 2 + 1$$

$$= 6$$

$$= \text{RHS}$$

4.

$$2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$$

Solution:

Consider

$$\text{L.H.S} = 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3}$$

So we get

$$= 2 \left\{ \sin \left(\pi - \frac{\pi}{4} \right) \right\}^2 + 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 2(2)^2$$

By further calculation

$$= 2 \left\{ \sin \frac{\pi}{4} \right\}^2 + 2 \times \frac{1}{2} + 8$$

It can be written as

$$= 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{RHS}$$

5. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Solution:

(i) $\sin 75^\circ$

It can be written as

$$= \sin (45^\circ + 30^\circ)$$

Using the formula $[\sin (x + y) = \sin x \cos y + \cos x \sin y]$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

Substituting the values

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

By further calculation

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

(ii) $\tan 15^\circ$

It can be written as

$$= \tan (45^\circ - 30^\circ)$$

Using formula

$$\tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

LOYAL Education

<https://loyaleducation.org>

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

Substituting the values

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

By further calculation

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

So we get

$$= \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$$

Prove the following:

6.

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Solution:

Consider LHS =

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

We can write it as

$$= \frac{1}{2} \left[2 \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) \right] + \frac{1}{2} \left[-2 \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) \right]$$

By further simplification

$$= \frac{1}{2} \left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\} \right]$$

$$+ \frac{1}{2} \left[\cos \left\{ \left(\frac{\pi}{4} - x \right) + \left(\frac{\pi}{4} - y \right) \right\} - \cos \left\{ \left(\frac{\pi}{4} - x \right) - \left(\frac{\pi}{4} - y \right) \right\} \right]$$

Using the formula

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

$$-2 \sin A \sin B = \cos (A + B) - \cos (A - B)$$

$$= 2 \times \frac{1}{2} \left[\cos \left\{ \left(\frac{\pi}{4} - x \right) + \left(\frac{\pi}{4} - y \right) \right\} \right]$$

We get

$$= \cos \left[\frac{\pi}{2} - (x + y) \right]$$

$$= \sin (x + y)$$

$$= \text{RHS}$$

7.

$$\frac{\tan \left(\frac{\pi}{4} + x \right)}{\tan \left(\frac{\pi}{4} - x \right)} = \left(\frac{1 + \tan x}{1 - \tan x} \right)^2$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\tan \left(\frac{\pi}{4} + x \right)}{\tan \left(\frac{\pi}{4} - x \right)}$$

By using the formula

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \text{and} \quad \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

So we get

$$= \frac{\left(\frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} \right)}{\left(\frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right)}$$

It can be written as

$$\begin{aligned} & \left(\frac{1 + \tan x}{1 - \tan x} \right) \\ &= \frac{\left(\frac{1 - \tan x}{1 + \tan x} \right)}{\left(\frac{1 + \tan x}{1 - \tan x} \right)} \\ &= \left(\frac{1 + \tan x}{1 - \tan x} \right)^2 \\ &= \text{RHS} \end{aligned}$$

8.

$$\frac{\cos(\pi + x) \cos(-x)}{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos(\pi + x) \cos(-x)}{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right)}$$

It can be written as

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

So we get

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$$= \cot^2 x$$

$$= \text{RHS}$$

9.

$$\cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right] = 1$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right]$$

It can be written as

$$= \sin x \cos x (\tan x + \cot x)$$

So we get

$$= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

$$= 1$$

= RHS

10. $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

Solution:

LHS = $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x$

By multiplying and dividing by 2

$$= \frac{1}{2} [2 \sin(n+1)x \sin(n+2)x + 2 \cos(n+1)x \cos(n+2)x]$$

Using the formula

$$- 2 \sin A \sin B = \cos(A+B) - \cos(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2} \left[\cos\{(n+1)x - (n+2)x\} - \cos\{(n+1)x + (n+2)x\} \right. \\ \left. + \cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \right]$$

By further calculation

$$= \frac{1}{2} \times 2 \cos\{(n+1)x - (n+2)x\}$$

$$= \cos(-x)$$

$$= \cos x$$

= RHS

11.

$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

Solution:

Consider

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

Using the formula

$$\begin{aligned}\cos A - \cos B &= -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right) \\ &= -2 \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\}\end{aligned}$$

So we get

$$= -2 \sin\left(\frac{3\pi}{4}\right) \sin x$$

It can be written as

$$= -2 \sin\left(\pi - \frac{\pi}{4}\right) \sin x$$

By further calculation

$$= -2 \sin \frac{\pi}{4} \sin x$$

Substituting the values

$$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\sqrt{2} \sin x$$

= RHS

$$\mathbf{12. \sin^2 6x - \sin^2 4x = \sin 2x \sin 10x}$$

Solution:

Consider

$$\text{L.H.S.} = \sin^2 6x - \sin^2 4x$$

Using the formula

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

So we get

$$= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)$$

By further calculation

$$= \left[2 \sin \left(\frac{6x+4x}{2} \right) \cos \left(\frac{6x-4x}{2} \right) \right] \left[2 \cos \left(\frac{6x+4x}{2} \right) \sin \left(\frac{6x-4x}{2} \right) \right]$$

We get

$$= (2 \sin 5x \cos x) (2 \cos 5x \sin x)$$

It can be written as

$$= (2 \sin 5x \cos 5x) (2 \sin x \cos x)$$

$$= \sin 10x \sin 2x$$

= RHS

$$\mathbf{13. \cos^2 2x - \cos^2 6x = \sin 4x \sin 8x}$$

Solution:

Consider

$$\text{L.H.S.} = \cos^2 2x - \cos^2 6x$$

Using the formula

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

So we get

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

By further calculation

$$= \left[2 \cos \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] \left[-2 \sin \left(\frac{2x+6x}{2} \right) \sin \left(\frac{2x-6x}{2} \right) \right]$$

We get

$$= [2 \cos 4x \cos (-2x)] [-2 \sin 4x \sin (-2x)]$$

It can be written as

$$= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$$

So we get

$$= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$$

$$= \sin 8x \sin 4x$$

= RHS

$$\mathbf{14. \sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x}$$

Solution:

Consider

$$\text{L.H.S.} = \sin 2x + 2 \sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2 \sin 4x$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$= \left[2 \sin \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] + 2 \sin 4x$$

By further simplification

$$= 2 \sin 4x \cos (-2x) + 2 \sin 4x$$

It can be written as

$$= 2 \sin 4x \cos 2x + 2 \sin 4x$$

Taking common terms

$$= 2 \sin 4x (\cos 2x + 1)$$

Using the formula

$$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$$

We get

$$= 2 \sin 4x (2 \cos^2 x)$$

$$= 4 \cos^2 x \sin 4x$$

= R.H.S.

$$15. \cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$$

Solution:

Consider

$$\text{LHS} = \cot 4x (\sin 5x + \sin 3x)$$

It can be written as

$$= \frac{\cos 4x}{\sin 4x} \left[2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right) \right]$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$= \left(\frac{\cos 4x}{\sin 4x} \right) [2 \sin 4x \cos x]$$

So we get

$$= 2 \cos 4x \cos x$$

Similarly

$$\text{R.H.S.} = \cot x (\sin 5x - \sin 3x)$$

It can be written as

$$= \frac{\cos x}{\sin x} \left[2 \cos \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right) \right]$$

Using the formula

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$= \frac{\cos x}{\sin x} [2 \cos 4x \sin x]$$

So we get

$$= 2 \cos 4x \cos x$$

Hence, LHS = RHS.

16.

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Solution:

Consider

$$\text{L.H.S} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

Using the formula

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\begin{aligned} &= \frac{-2 \sin\left(\frac{9x+5x}{2}\right) \cdot \sin\left(\frac{9x-5x}{2}\right)}{2 \cos\left(\frac{17x+3x}{2}\right) \cdot \sin\left(\frac{17x-3x}{2}\right)} \end{aligned}$$

By further calculation

$$= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x}$$

So we get

$$= -\frac{\sin 2x}{\cos 10x}$$

= RHS

17.

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

Using the formula

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\begin{aligned} &= \frac{2 \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)}{2 \cos\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)} \end{aligned}$$

By further calculation

$$= \frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x}$$

So we get

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x$$

= RHS

18.

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

Using the formula

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$= \frac{2 \cos \left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right)}{2 \cos \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right)}$$

By further calculation

$$= \frac{\sin \left(\frac{x-y}{2} \right)}{\cos \left(\frac{x-y}{2} \right)}$$

So we get

$$= \tan \left(\frac{x-y}{2} \right)$$

= RHS

19.

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

Using the formula

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2 \sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}{2 \cos\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}$$

By further calculation

$$= \frac{\sin 2x}{\cos 2x}$$

So we get

$$= \tan 2x$$

= RHS

20.

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

Using the formula

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos^2 A - \sin^2 A = \cos 2A$$

$$= \frac{2 \cos\left(\frac{x+3x}{2}\right) \sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

By further calculation

$$= \frac{2 \cos 2x \sin(-x)}{-\cos 2x}$$

So we get

$$= -2(-\sin x)$$

$$= 2 \sin x$$

= RHS

21.

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution:

Consider

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

It can be written as

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

Using the formula

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$= \frac{2 \cos\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2 \sin\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

By further calculation

$$= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x}$$

So we get

$$= \frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)}$$

$$= \cot 3x$$

$$= \text{RHS}$$

$$22. \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

Solution:

Consider

$$\text{LHS} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$

It can be written as

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$$

Using the formula

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x)$$

So we get

$$= \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1$$

$$= \text{RHS}$$

23.

$$\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

Solution:

Consider

$$\text{LHS} = \tan 4x = \tan 2(2x)$$

By using the formula

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$= \frac{2 \tan 2x}{1 - \tan^2 (2x)}$$

It can be written as

$$= \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

By further simplification

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

Taking LCM

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

On further simplification

$$= \frac{4 \tan x (1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4 \tan^2 x}$$

We get

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$$

It can be written as

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

= RHS

$$\mathbf{24. \cos 4x = 1 - 8\sin^2 x \cos^2 x}$$

Solution:

Consider

$$\text{LHS} = \cos 4x$$

We can write it as

$$= \cos 2(2x)$$

Using the formula $\cos 2A = 1 - 2 \sin^2 A$

$$= 1 - 2 \sin^2 2x$$

Again by using the formula $\sin 2A = 2 \sin A \cos A$

$$= 1 - 2(2 \sin x \cos x)^2$$

So we get

$$= 1 - 8 \sin^2 x \cos^2 x$$

= R.H.S.

$$25. \cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

Solution:

Consider

$$\text{L.H.S.} = \cos 6x$$

It can be written as

$$= \cos 3(2x)$$

Using the formula $\cos 3A = 4 \cos^3 A - 3 \cos A$

$$= 4 \cos^3 2x - 3 \cos 2x$$

Again by using formula $\cos 2x = 2 \cos^2 x - 1$

$$= 4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)]$$

By further simplification

$$= 4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6 \cos^2 x + 3$$

We get

$$= 4 [8 \cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$$

By multiplication

$$= 32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$$

On further calculation

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

= R.H.S.

Exercise 3.4 PAGE: 78

Find the principal and general solutions of the following equations:

1. $\tan x = \sqrt{3}$

Solution:

It is given that

$$\tan x = \sqrt{3}$$

We know that

$$\tan \frac{\pi}{3} = \sqrt{3}$$

It can be written as

$$\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right)$$

So we get

$$= \tan \frac{\pi}{3} = \sqrt{3}$$

Hence, the principal solutions are $x = \pi/3$ and $4\pi/3$

$$\tan x = \tan \frac{\pi}{3}$$

We get

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

2. $\sec x = 2$

Solution:

It is given that

$$\sec x = 2$$

We know that

$$\sec \frac{\pi}{3} = 2$$

It can be written as



LOYAL Education
<https://loyaleducation.org>

$$\sec \frac{5\pi}{3} = \sec \left(2\pi - \frac{\pi}{3} \right)$$

So we get

$$= \sec \frac{\pi}{3} = 2$$

Hence, the principal solutions are $x = \pi/3$ and $5\pi/3$.

$$\sec x = \sec \frac{\pi}{3}$$

We know that $\sec x = 1/\cos x$

$$\cos x = \cos \frac{\pi}{3}$$

So we get

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

3. $\cot x = -\sqrt{3}$

Solution:

It is given that

$$\cot x = -\sqrt{3}$$

We know that

$$\cot \frac{\pi}{6} = \sqrt{3}$$

It can be written as

$$\cot \left(\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

And

$$\cot \left(2\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

So we get

$$\cot \frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot \frac{11\pi}{6} = -\sqrt{3}$$

Hence, the principal solutions are $x = 5\pi/6$ and $11\pi/6$.

$$\cot x = \cot \frac{5\pi}{6}$$

We know that $\cot x = 1/\tan x$

$$\tan x = \tan \frac{5\pi}{6}$$

So we get

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}$$

4. cosec $x = -2$

Solution:

It is given that

$$\operatorname{cosec} x = -2$$

We know that

$$\operatorname{cosec} \frac{\pi}{6} = 2$$

It can be written as

$$\operatorname{cosec} \left(\pi + \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

And

$$\operatorname{cosec} \left(2\pi - \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

So we get

$$\operatorname{cosec} \frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec} \frac{11\pi}{6} = -2$$

Hence, the principal solutions are $x = 7\pi/6$ and $11\pi/6$.

$$\operatorname{cosec} x = \operatorname{cosec} \frac{7\pi}{6}$$



education
leducation.org

We know that $\operatorname{cosec} x = 1/\sin x$

$$\sin x = \sin \frac{7\pi}{6}$$

So we get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Find the general solution for each of the following equations:

5. $\cos 4x = \cos 2x$

Solution:

It is given that

$$\cos 4x = \cos 2x$$

We can write it as

$$\cos 4x - \cos 2x = 0$$

Using the formula

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

We get

$$-2 \sin \left(\frac{4x+2x}{2} \right) \sin \left(\frac{4x-2x}{2} \right) = 0$$

By further simplification

$$\sin 3x \sin x = 0$$

We can write it as

$$\sin 3x = 0 \text{ or } \sin x = 0$$

By equating the values

$$3x = n\pi \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$$

We get

$$x = n\pi/3 \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$$

6. $\cos 3x + \cos x - \cos 2x = 0$

Solution:

It is given that

$$\cos 3x + \cos x - \cos 2x = 0$$

We can write it as

$$2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$$

Using the formula

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

We get

$$2 \cos 2x \cos x - \cos 2x = 0$$

By further simplification

$$\cos 2x (2 \cos x - 1) = 0$$

We can write it as

$$\cos 2x = 0 \text{ or } 2 \cos x - 1 = 0$$

$$\cos 2x = 0 \text{ or } \cos x = 1/2$$

By equating the values

$$2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

We get

$$x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$\mathbf{7. \sin 2x + \cos x = 0}$$

Solution:

It is given that

$$\sin 2x + \cos x = 0$$

We can write it as

$$2 \sin x \cos x + \cos x = 0$$

$$\cos x (2 \sin x + 1) = 0$$

$$\cos x = 0 \text{ or } 2 \sin x + 1 = 0$$

$$\text{Let } \cos x = 0$$

$$\cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in \mathbb{Z}$$

$$2 \sin x + 1 = 0$$

So we get

$$\sin x = \frac{-1}{2} = -\sin \frac{\pi}{6}$$

We can write it as

$$= \sin \left(\pi + \frac{\pi}{6} \right) = \sin \left(\pi + \frac{\pi}{6} \right)$$

$$= \sin \frac{7\pi}{6}$$

We get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$(2n+1)\frac{\pi}{2} \text{ or } n\pi + (-1)^n \frac{7\pi}{6}, n \in \mathbb{Z}$$

8. $\sec^2 2x = 1 - \tan 2x$

Solution:

It is given that

$$\sec^2 2x = 1 - \tan 2x$$

We can write it as

$$1 + \tan^2 2x = 1 - \tan 2x$$

$$\tan^2 2x + \tan 2x = 0$$

Taking common terms

$$\tan 2x (\tan 2x + 1) = 0$$

Here

$$\tan 2x = 0 \text{ or } \tan 2x + 1 = 0$$

$$\text{If } \tan 2x = 0$$

$$\tan 2x = \tan 0$$

We get

$$2x = n\pi + 0, \text{ where } n \in \mathbb{Z}$$

$$x = n\pi/2, \text{ where } n \in \mathbb{Z}$$

$$\tan 2x + 1 = 0$$

We can write it as

$$\tan 2x = -1$$

So we get

$$= -\tan \frac{\pi}{4} = \tan \left(\pi - \frac{\pi}{4} \right)$$

$$= \tan \frac{3\pi}{4}$$

Here

$$2x = n\pi + 3\pi/4, \text{ where } n \in \mathbb{Z}$$

$$x = n\pi/2 + 3\pi/8, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is $n\pi/2$ or $n\pi/2 + 3\pi/8, n \in \mathbb{Z}$.

$$\mathbf{9. \sin x + \sin 3x + \sin 5x = 0}$$

Solution:

It is given that

$$\sin x + \sin 3x + \sin 5x = 0$$

We can write it as

$$(\sin x + \sin 5x) + \sin 3x = 0$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\left[2 \sin \left(\frac{x+5x}{2} \right) \cos \left(\frac{x-5x}{2} \right) \right] + \sin 3x = 0$$

By further calculation

$$2 \sin 3x \cos (-2x) + \sin 3x = 0$$

It can be written as

$$2 \sin 3x \cos 2x + \sin 3x = 0$$

By taking out the common terms

$$\sin 3x (2 \cos 2x + 1) = 0$$

Here

$$\sin 3x = 0 \text{ or } 2 \cos 2x + 1 = 0$$

$$\text{If } \sin 3x = 0$$

$$3x = n\pi, \text{ where } n \in \mathbb{Z}$$

We get

$$x = n\pi/3, \text{ where } n \in \mathbb{Z}$$

$$\text{If } 2 \cos 2x + 1 = 0$$

$$\cos 2x = -1/2$$

By further simplification

$$= -\cos \pi/3$$

$$= \cos (\pi - \pi/3)$$

So we get

$$\cos 2x = \cos 2\pi/3$$

Here

$$2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Dividing by 2

$$x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Miscellaneous Exercise page: 81

Prove that:

1.

$$2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$$

Solution:

LOYALEducation
<https://loyaleducation.org>

$$\text{L.H.S.} = 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}$$

Using the formula

$$\cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

So we get

$$= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \left[\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2} \right] \cos \left[\frac{\frac{3\pi}{13} - \frac{5\pi}{13}}{2} \right]$$

By further calculation

$$= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \left(\frac{-\pi}{13} \right)$$

We get

$$= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \frac{\pi}{13}$$

Taking out the common terms

$$= 2 \cos \frac{\pi}{13} \left[\cos \frac{9\pi}{13} + \cos \frac{4\pi}{13} \right]$$

It can be written as

$$= 2 \cos \frac{\pi}{13} \left[2 \cos \left(\frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2} \right) \cos \left(\frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2} \right) \right]$$

On further calculation

$$= 2 \cos \frac{\pi}{13} \left[2 \cos \frac{\pi}{2} \cos \frac{5\pi}{26} \right]$$

We get

$$= 2 \cos \frac{\pi}{13} \times 2 \times 0 \times \cos \frac{5\pi}{26}$$

$$= 0$$

$$= \text{RHS}$$

$$2. (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$$

Solution:

Consider

$$\text{LHS} = (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

By further calculation

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$$

Taking out the common terms

$$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$$

Using the formula

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$= \cos (3x - x) - \cos 2x$$

So we get

$$= \cos 2x - \cos 2x$$

$$= 0$$

$$= \text{RHS}$$

3.

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$$

Solution:

Consider

$$\text{LHS} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

By expanding using formula we get

$$= \cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$$

Grouping the terms

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2 (\cos x \cos y - \sin x \sin y)$$

Using the formula $\cos (A + B) = (\cos A \cos B - \sin A \sin B)$

$$= 1 + 1 + 2 \cos (x + y)$$

By further calculation

$$= 2 + 2 \cos (x + y)$$

Taking 2 as common

$$= 2 [1 + \cos (x + y)]$$

From the formula $\cos 2A = 2 \cos^2 A - 1$

$$= 2 \left[1 + 2 \cos^2 \left(\frac{x+y}{2} \right) - 1 \right]$$

We get

$$= 4 \cos^2 \left(\frac{x+y}{2} \right)$$

$$= \text{RHS}$$

4.

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \frac{x-y}{2}$$

Solution:

$$\text{LHS} = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

By expanding using formula

$$= \cos^2 x + \cos^2 y - 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$$

Grouping the terms

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2 (\cos x \cos y + \sin x \sin y)$$

Using the formula $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$= 1 + 1 - 2 [\cos (x - y)]$$

By further calculation

$$= 2 [1 - \cos (x - y)]$$

From formula $\cos 2A = 1 - 2 \sin^2 A$

$$= 2 \left[1 - \left\{ 1 - 2 \sin^2 \left(\frac{x - y}{2} \right) \right\} \right]$$

We get

$$= 4 \sin^2 \left(\frac{x - y}{2} \right)$$

= RHS

$$5. \sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$$

Solution:

Consider

$$\text{LHS} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

Grouping the terms

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A + B}{2} \right) \cdot \cos \left(\frac{A - B}{2} \right)$$

So we get

$$= 2 \sin \left(\frac{x + 5x}{2} \right) \cdot \cos \left(\frac{x - 5x}{2} \right) + 2 \sin \left(\frac{3x + 7x}{2} \right) \cos \left(\frac{3x - 7x}{2} \right)$$

By further calculation

$$= 2 \sin 3x \cos (-2x) + 2 \sin 5x \cos (-2x)$$

We get

$$= 2 \sin 3x \cos 2x + 2 \sin 5x \cos 2x$$

Taking out the common terms

$$= 2 \cos 2x [\sin 3x + \sin 5x]$$

Using the formula we can write it as

$$= 2 \cos 2x \left[2 \sin \left(\frac{3x+5x}{2} \right) \cdot \cos \left(\frac{3x-5x}{2} \right) \right]$$

We get

$$= 2 \cos 2x [2 \sin 4x \cdot \cos (-x)]$$

$$= 4 \cos 2x \sin 4x \cos x$$

= RHS

6.

$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Solution:

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right), \quad \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$= \frac{\left[2 \sin \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2 \sin \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]}{\left[2 \cos \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2 \cos \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]}$$

By further calculation

$$= \frac{[2 \sin 6x \cdot \cos x] + [2 \sin 6x \cdot \cos 3x]}{[2 \cos 6x \cdot \cos x] + [2 \cos 6x \cdot \cos 3x]}$$

Taking out the common terms

$$= \frac{2 \sin 6x [\cos x + \cos 3x]}{2 \cos 6x [\cos x + \cos 3x]}$$

We get

$$= \tan 6x$$

= RHS

7.

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

Solution:

$$\text{LHS} = \sin 3x + \sin 2x - \sin x$$

It can be written as

$$= \sin 3x + (\sin 2x - \sin x)$$

Using the formula

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$= \sin 3x + \left[2 \cos \left(\frac{2x+x}{2} \right) \sin \left(\frac{2x-x}{2} \right) \right]$$

By further simplification

$$= \sin 3x + \left[2 \cos \left(\frac{3x}{2} \right) \sin \left(\frac{x}{2} \right) \right]$$

$$= \sin 3x + 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

Using formula $\sin 2A = 2 \sin A \cos B$

$$= 2 \sin \frac{3x}{2} \cdot \cos \frac{3x}{2} + 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

Taking out the common terms

$$= 2 \cos \left(\frac{3x}{2} \right) \left[\sin \left(\frac{3x}{2} \right) + \sin \left(\frac{x}{2} \right) \right]$$

From the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$= 2 \cos \left(\frac{3x}{2} \right) \left[2 \sin \left\{ \frac{\left(\frac{3x}{2} \right) + \left(\frac{x}{2} \right)}{2} \right\} \cos \left\{ \frac{\left(\frac{3x}{2} \right) - \left(\frac{x}{2} \right)}{2} \right\} \right]$$

By further simplification

$$= 2 \cos \left(\frac{3x}{2} \right) \cdot 2 \sin x \cos \left(\frac{x}{2} \right)$$

We get

$$= 4 \sin x \cos \left(\frac{x}{2} \right) \cos \left(\frac{3x}{2} \right)$$

= RHS

8. Find $\sin x/2$, $\cos x/2$ and $\tan x/2$ in each of the following:

$$\tan x = -\frac{4}{3}, x \text{ in quadrant II}$$

Solution:

It is given that

x is in quadrant II

$$\frac{\pi}{2} < x < \pi$$

Dividing by 2

$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Hence, $\sin x/2$, $\cos x/2$ and $\tan x/2$ are all positive.

$$\tan x = -\frac{4}{3}$$

From the formula $\sec^2 x = 1 + \tan^2 x$

Substituting the values

$$\sec^2 x = 1 + (-4/3)^2$$

We get

$$= 1 + 16/9 = 25/9$$

Here

$$\cos^2 x = \frac{9}{25}$$

$$\cos x = \pm \frac{3}{5}$$

Here x is in quadrant II, $\cos x$ is negative.

$$\cos x = -3/5$$

From the formula



Education

<https://loyaleducation.org>

$$\cos x = 2 \cos^2 \frac{x}{2} - 1$$

Substituting the values

$$\frac{-3}{5} = 2 \cos^2 \frac{x}{2} - 1$$

By further calculation

$$2 \cos^2 \frac{x}{2} = 1 - \frac{3}{5}$$

$$2 \cos^2 \frac{x}{2} = \frac{2}{5}$$

$$\cos^2 \frac{x}{2} = \frac{1}{5}$$

We get

$$\cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$

$$\cos \frac{x}{2} = \frac{\sqrt{5}}{5}$$

From the formula

$$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$$

Substituting the value

$$\sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}} \right)^2 = 1$$

By further calculation

$$\sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

We get

$$\sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

$$\sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$$

Here

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Hence, the respective values of $\sin x/2$, $\cos x/2$ and $\tan x/2$ are

$$\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \text{ and } 2$$

9. $\cos x = -1/3$, x in quadrant III

Solution:

It is given that

x is in quadrant III

$$\pi < x < \frac{3\pi}{2}$$

Dividing by 2

$$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Hence, $\cos x/2$ and $\tan x/2$ are negative where $\sin x/2$ is positive.

$$\cos x = -\frac{1}{3}$$

From the formula $\cos x = 1 - 2 \sin^2 x/2$

We get

$$\sin^2 x/2 = (1 - \cos x)/2$$

Substituting the values

$$\sin^2 \frac{x}{2} = \frac{1 - \left(-\frac{1}{3}\right)}{2} = \frac{\left(1 + \frac{1}{3}\right)}{2}$$

We get

$$\frac{4}{3} = \frac{2}{2} = \frac{2}{3}$$

Here

$$\sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Using the formula

$$\cos x = 2 \cos^2 \frac{x}{2} - 1$$

We get

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

Substituting the values

$$= \frac{1 + \left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2}$$

$$= \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

We get

$$\cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$$

By further calculation

$$\cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

Here

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Therefore, the respective values of $\sin x/2$, $\cos x/2$ and $\tan x/2$ are

$$\frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}, \text{ and } -\sqrt{2}$$

10. $\sin x = 1/4$, x in quadrant II

Solution:

It is given that

x is in quadrant II

$$\frac{\pi}{2} < x < \pi$$

Dividing by 2

$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Hence, $\sin x/2$, $\cos x/2$ and $\tan x/2$ are positive.

$$\sin x = \frac{1}{4}$$

From the formula $\cos^2 x = 1 - \sin^2 x$

We get

$$\cos^2 x = 1 - (1/4)^2$$

Substituting the values

$$\cos^2 x = 1 - 1/16 = 15/16$$

We get

$$\cos x = -\frac{\sqrt{15}}{4}$$

Here

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

Substituting the values

$$= \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 + \sqrt{15}}{8}$$

We get

$$\sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

Multiplying and dividing by 2

$$= \sqrt{\frac{4 + \sqrt{15}}{8} \times \frac{2}{2}}$$

By further calculation

$$= \sqrt{\frac{8 + 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 + 2\sqrt{15}}}{4}$$

Here

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

By substituting the values

$$= \frac{1 + \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 - \sqrt{15}}{8}$$

We get

$$\cos \frac{x}{2} = \sqrt{\frac{4 - \sqrt{15}}{8}}$$

By multiplying and dividing by 2

$$= \sqrt{\frac{4 - \sqrt{15}}{8} \times \frac{2}{2}}$$

It can be written as

$$= \sqrt{\frac{8 - 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 - 2\sqrt{15}}}{4}$$

We know that

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

Substituting the values

$$\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

By multiplying and dividing the terms

$$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}$$

We get

$$= \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2}$$

$$= 4 + \sqrt{15}$$

Therefore, the respective values of $\sin x/2$, $\cos x/2$ and $\tan x/2$ are

$$\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8-2\sqrt{15}}}{4} \text{ and } 4 + \sqrt{15}$$

LOYAL Education
<https://loyaleducation.org>